### Industrial organisation

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# Preface

This is a live document, and is full of gaps, mistakes, typos etc.

# Part I Monopoly pricing

# Perfect competition

- 1.1 Perfect competition
- 1.1.1 Perfect competition
- 1.1.2 Hotelling's lemma
- 1.1.3 Short-term supply function
- 1.1.4 Long-term supply function
- 1.1.5 Price elasticity of supply

# Price discrimination and two-part tariffs

### 2.1 Price discrimination

### 2.1.1 Barriers to resale

### 2.1.2 First-degree price discrimination

perfect. have complete knowledge of customers Customer pays reservation price. Requires firm to know reservation price.

### 2.1.3 Second-degree price discrimination

discounts for quantity. eg industrial orders. have no knowledge of customers Price varies by quantity ordered.

### 2.1.4 Third-degree price discrimination

different for different groups. eg student discount. have knowldege of groups Different prices to different customer segments.

#### 2.1.5 Two-part tariff

Subscription services are a type of two-part tariff.

# Intertemporal monopoly pricing and the Coase and Pacman conjectures

### 3.1 Introduction

### 3.1.1 The Coase conjecture

Context of intertemporal pricing.

A monopolist can price discriminate by offering different prices at different times.

Coase conjecture is that under certain assumptions, this doesn't work, and the monopolist will profit maximise by offering low price across time.

### 3.1.2 The Pacman conjecture

Related to Coase conjecture but opposite conclusion.

Monopolist can initially set prices at highest level then slowly lower them, eating total surplus (hence "pacman").

Relies on infinite time horizon.

Both Coase conjecture and Pacman conjecture are in the context of durable goods.

# Part II

# Horizontal homogeneous

# Pricing homogeneous goods with Cournot and Bertrand model

### 4.1 The economic profit function

### 4.1.1 Profit

The profit of a firm is the difference between revenue and costs.

 $\pi = pq - c$ 

Where q is the amount producted, and p is the price, and c is a function of production.

### 4.1.2 Maximising profit

 $\pi = pq - c$ 

The firm's production q affects the market price p.

$$\begin{split} &\frac{\delta\pi}{\delta q} = \frac{\delta}{\delta q} [pq-c] \\ &\frac{\delta\pi}{\delta q} = p + q \frac{\delta p}{\delta q} - \frac{\delta c}{\delta q} \end{split}$$

The firm chooses q to maximise profits, so  $\frac{\delta \pi}{\delta q} = 0$ .

$$0 = p + q \frac{\delta p}{\delta q} - \frac{\delta c}{\delta q}$$

$$p + q\frac{\delta p}{\delta q} = \frac{\delta c}{\delta q}$$

The right side is marginal costs (MC), the left is marginal revenue.

$$p[1 + \frac{q}{p}\frac{\delta p}{\delta q}] = MC$$

We know that the price elasticity of demand is:  $\epsilon = \frac{p}{q} \frac{\delta q}{\delta p}$ 

So we have:

$$p[1 + \frac{1}{\epsilon}] = MC$$
$$p = \frac{\epsilon}{1 + \epsilon}MC$$

This can be plugged into the demand function to get the quantity supplied.

### 4.1.3 Intensive and extensive margins

$$revenue = pq$$
$$MR = p + q \frac{\delta p}{\delta q}$$

p is the extensive margin.

 $q\frac{\delta p}{\delta q}$  is the (negative) intensive margin.

monopoly pricing. when lower prices, gain money on extensive margin. lose money on intensive margin.

### 4.2 Cournot competition

### 4.2.1 Cournot competition

With competition, the elasticity of demand refers to the whole market, not just a single producer. Instead we have:

$$\epsilon = \frac{p}{Q} \frac{\delta Q}{\delta p}$$
$$Q = \sum_{j} q_{j}$$
We now go

We now get:

$$\begin{split} p[1 + \frac{q}{Q} \frac{\delta Q}{\delta q} \frac{Q}{p} \frac{\delta p}{\delta Q}] &= MC\\ p[1 + \frac{\mu}{\epsilon}] &= MC \end{split}$$

 $p = MC \frac{\epsilon}{\epsilon + \mu}$ 

Using the firm's size elasticity:  $\mu = \frac{q}{Q} \frac{\delta Q}{\delta q}$ With monopoly  $\mu = 1$ .

### 4.2.2 Horizontal mergers

Scale benefits.

### 4.3 Bertrand competition

### 4.3.1 Bertrand competition

Each player decides what price to sell at.

Firms who price above the lowest have no sales. Prices converge to cost.

### 4.4 Horizontal single good

- 4.4.1 Entry Rate (ER)
- 4.4.2 Entrant Relative Size (ERS)
- 4.4.3 Exiter Relative Size (XRS)
- 4.4.4 Exit Rate (XR)
- 4.4.5 Limit pricing
- 4.4.6 Bresnahan-Reiss entry model
- 4.5 Natural sources of market power
- 4.5.1 Returns to scale
- 4.5.2 Heterogeneous efficiency
- 4.6 Using marginal cost data

### 4.6.1 Monopoly

In the monopoly model we have:

$$\begin{split} \pi &= pq-c \\ p[1+\frac{q}{p}\frac{\delta p}{\delta q}] = MC \end{split}$$

The price elasticity of demand is:  $\epsilon = \frac{p}{q} \frac{\delta q}{\delta p}$ 

$$p[1 + \frac{1}{\epsilon}] = MC$$
$$\frac{1}{\epsilon} = \frac{MC}{p} - 1$$
$$\frac{p - MC}{p} = -\frac{1}{\epsilon}$$

### 4.6.2 The Lerner index

The Lerner index is:  $\underline{p - MC}$ 

### 4.6.3 Cournot model

With competition, the elasticity of demand refers to the whole market, not just a single producer. Instead we have:

$$\epsilon = \frac{p}{Q} \frac{\delta Q}{\delta p}$$
$$Q = \sum_{j} q_{j}$$

We now get:

$$p[1 + \frac{q}{Q}\frac{\delta Q}{\delta q}\frac{Q}{p}\frac{\delta p}{\delta Q}] = MC$$
$$p[1 + \frac{\mu}{\epsilon}] = MC$$

Using the firm's size elasticity:  $\mu = \frac{q}{Q} \frac{\delta Q}{\delta q}$ 

With monopoly this is:

$$\mu = 1$$

In this model this is:

$$\frac{p-MC}{p} = -\frac{\mu}{\epsilon}$$

### 4.7 Measuring market concentration

### 4.7.1 Herfindahl–Hirschman Index (HHI)

The index is the sum of each firm's market share squared.

For a monopolist this is 1, for a completely competitive market it is 0, or  $\frac{1}{n}$ .

 $H = \sum_{i=1}^n s_i^2$ 

If this is just done with the largest firms, the results will be similar to the result for the whole market, due to the quadratic element.

To normalise this between 0 and 1 we can use:

$$H* = \frac{H - \frac{1}{n}}{1 - \frac{1}{n}}$$

1

### 4.7.2 Concentration ratio

Proportion of output from given firms.

For example  $CR_5$  is the proportion of output from the 5 largest producers.

#### 4.7.3 Lerner index

Marginal profit. If high it suggests existing power prevents it from raising output.

$$L = \frac{P - MC}{P}$$

From 0 to 1.

4.7.4 Pivotal supplier index (PSI)

4.7.5 Residual Supply Index (RSI)

# Pricing in repeating rounds

### 5.1 Pricing in repeated rounds

### 5.1.1 Stackleberg competition

Sequential Cournot competition.

There is a first-mover advantage.

- 5.1.2 Explicit and tacit collusion
- 5.1.3 Monitoring and enforcing collusion

# Horizontal mergers and merger simulation

- 6.1 Horizontal single good
- 6.1.1 Tying

# **Econometrics of production**

7.1	Measuring	heterogeneous	efficiency

- 7.1.1 Stochastic frontier analysis
- 7.1.2 Data envelopment analysis

# Part III

# Horizontal multi good

# Pricing heterogeneous goods

### 8.1 Trade

### 8.1.1 Horizontal competition

Assured and contestable demand So buyer wants 100 from incumbent, and 50 more. First 100 are assured. 50 are contestable. Cournot, bertrand mean entrant gets something However rebates from incumbant blocks entrant. eg reduce unit price for all if sales above certain amount

### 8.2 Product differentiation

### 8.2.1 Hotelling competition

Compete on quality, features, on an axis. For example 2 shops in equilibrium are next to each other on road, to capture both on either side.

### 8.2.2 Vertical differentiation

quality and price. everyone prefers high quality, but different preferences for tradeoff

### 8.2.3 Horizontal differentiation

different preferences for characteristics. eg car colour

### 8.2.4 Bertrand competition with differentiated products

 $q_1 = a - b_1 p_1 + b_2 p_2 \ q_2 = a - b_1 p_2 + b_2 p_1$ 

### 8.3 Pricing multiple products

### 8.3.1 Bertrand competition with differentiated products

Can have competing products.

### 8.4 New product types

### 8.4.1 Identifying new products

eg car reach consumer needs in different ways, different way of looking at their demand function

### 8.4.2 Selling to customers

retail? to businesses (cold calling?)

### 8.4.3 Maintaining market share

# Pricing renewing customers

### Market definition

### 10.1 Market definition

### 10.1.1 SSNIP test

Small but significant and non-transitory increase in price

Would this cause customers to move elsewhere?

If we have demand at different prices we can estimate whether it is worth monopolising the market.

Because this lack of motivation may be because of competitor goods, we can include these to see if that market is worth monopolising.

For example: hiking up the price of one good may not be profitable. Therefore that is not a relevant market (though it could be!). Then we can see if hiking up the price of that good, and others, is profitable. If so, then it is a relevant market.

Note:

High elasticity may be because monopoly power is already being exerted

Identify smallest market where a monopolist could increas price profitably

How to do test? interview customers about whether increase in price would negatively affect them. want to know if they could switch.

If could switch at price rise of say 5

Can be used to estimate elasticity of demand

We can also look at the cost impact from cutting units. if high variable, then more appealing

We can expand to include substitutes. if substitutes make worth monopolisign, then merger can be concerning.

### 10.1.2 Market definition

#### Defining the relevant market

We want to see what market the monopolist can exert a profitable increase in price. This my not be all of their offerings.

The relevant market includes the good offered by the monopolist, along with relevant competitors in supply and demand

#### What is the product?

If we are considering a commodity it is easy to see that, say, steel supplied by one firm is comparable to that supplied by another. For other goods this is more complex.

For example, does Google provide search services, making it highly dominant? Or does it in fact provide advertising services for a small number of searches aimed at purchases? In the latter case it is a closer good to Amazon or Ebay.

#### Supply side substitution

If a price rise from a firm caused other firms to increase supply, this is relevant.

Would others be able to raise output?

Would others be able to enter the market?

#### Demand side substitution

If a price rise from a firm causes buyers to react, price increases will be less rewarding for the firm.

Assessment: Price elasticity of demand

#### Geographic market

May be many players but fewer locally. Threat of entry may still be a key motivation for price setting. Supply side substitution.

#### Vertical integration

Benefits: no double mark up.

2 monopolists both exert monopoly power, more deadweight loss. Integration solves this.

Contract theory argument. Don't want to be held if need change

Costs: can keep out downstream competitors

Facilitating collusion? Vertical integration allows upstream to monitor downstream price from their customer.

Restoring monopoly power?

Problem: to what extent can monopoly upstream abuse their power? One option is high price, but they could also do 2 part tariffs

But 2 part tariffs are unstable, as there is an incentive for the provider to offer the last downstream one a lower marginal cost.

Vertical integration then restores this power.

# Part IV

# Monopsony

# $\mathbf{Part}~\mathbf{V}$

# Vertical

# Vertical mergers and merger simulation

### 11.1 Vertical

11.1.1 Restriction of access

### 11.1.2 Vertical mergers

Remove double margin. Prevent supply to competitors.

### 11.1.3 Vertical production and margin squeeze

# Part VI SORT

# Estimating structural- and reduced-form models

### 12.1 The problem with estimating structural models

### 12.1.1 Structural supply and demand functions

Supply:

 $Q_s = \alpha_1 + \beta_1 P + \gamma_1 I + \epsilon_1$ 

Demand:

 $Q_d = \alpha_2 + \beta_2 P + \gamma_2 I + \epsilon_2$ 

Can't estimate because the equations are simulataneous.

To estimate,  $cov(P, \epsilon_1)$  needs to be 0, but what is it?

 $cov(P,\epsilon_1) = E[(P - E[P])(\epsilon_1 - E[\epsilon_1])] \ cov(P,\epsilon_1) = E[(P - E[P])(\epsilon_1 - E[\epsilon_1])]$ 

### 12.1.2 The identification problem

### 12.2 Estimating reduced-form models

### 12.2.1 Structural supply and demand functions

Supply:

 $Q_s = \alpha_1 + \beta_1 P + \gamma_1 I + \epsilon_1$ Demand:  $Q_d = \alpha_2 + \beta_2 P + \gamma_2 I + \epsilon_2$ 

Can't estimate because the equations are simulataneous.

### 12.2.2 Reduced form equations

Where supply is demand.

$$Q_{s} = Q_{d}$$

$$\alpha_{1} + \beta_{1}P + \gamma_{1}I + \epsilon_{1} = \alpha_{2} + \beta_{2}P + \gamma_{2}I + \epsilon_{2}$$

$$(\alpha_{1} - \alpha_{2}) + (\beta_{1} - \beta_{2})P + (\gamma_{1} - \gamma_{2})I + (\epsilon_{1} - \epsilon_{2}) = 0$$

$$(\beta_{1} - \beta_{2})P = -(\alpha_{1} - \alpha_{2}) - (\gamma_{1} - \gamma_{2})I - (\epsilon_{1} - \epsilon_{2})$$

$$P = -\frac{\alpha_{1} - \alpha_{2}}{\beta_{1} - \beta_{2}} - \frac{\gamma_{1} - \gamma_{2}}{\beta_{1} - \beta_{2}}I - \frac{\epsilon_{1} - \epsilon_{2}}{\beta_{1} - \beta_{2}}$$

We can construct something similar for Q. The results are reduced-form parameters with reduced-form errors.

### 12.2.3 More on reduced form

reduced form for perfect competition, and imperfect

issue is: supply function only defined for perfect competition.

how do you get equilibrium otherwise? what are the other reduced form equations? strucutral?

### 12.3 Using 2-stage OLS

### 12.3.1 Using IVs

Let's say the demand function is:

 $Q_d = \alpha + \beta P + \epsilon$ 

How can we estimate this?

OLS will give biased results if P is correlated with  $\epsilon$ .

We can estimate if we have an instrumental variable for P.

### 12.3.2 Power of IVs

need variation. if factor **v** important, little price movelemtn. may be hard to esimate that fact.

### 12.3.3 Locality of elasticity

we measure elsaticities as they are. there may be existing competition barriers which cause current substitution. without current monopolies, there may be more unique markets.

eg monopolist has product with sbustitutve, but it is only substitute because the monopolist has kept the price so high.

### 12.4 Examples of IVs

- 12.4.1 Cost data from firms
- 12.4.2 Weather
- 12.4.3 Exogeneous cost increases

### 12.4.4 Product characteristics

We can use the characteristics of products, and other products.

### 12.4.5 Price increases in other geographies

Assume only correlated with marginal cost in other geography.

### 12.4.6 Price changes of substitutes/complements

# Part VII

# Auctions

### Auction theory

### 13.1 Introduction

13.1.1 Introduction

### 13.2 English auctions

### 13.2.1 English auctions

In an English auction bidders make ascending bids on a product, each bid needing to be higher than the last.

These are open auctions. Each bid is visible to all participants.

The price paid is the final bid.

#### 13.2.2 English auctions

In an English auction bidders make ascending bids on a product, each bid needing to be higher than the last.

These are open auctions. Each bid is visible to all participants.

The price paid is the final bid.

### 13.3 Dutch auctions

### 13.3.1 Dutch auctions

In a Dutch auction the auctioneer decreases the price until a single bid is received.

These are open auctions. Each bid is visible to all participants.

The price paid is the bid.

### 13.4 Blind auctions

### 13.4.1 Blind auctions

Blind auctions are sealed-bid auctions. Participants cannot view other bids. The price paid is the highest bid.

### 13.5 Vickrey auctions

### 13.5.1 Vickrey auctions

Vickrey are sealed-bid auctions. Participants cannot view other bids. The price paid is the second-highest bid.

# Advanced auction theory

14.1 Auctions for multiple items

# Part VIII

# Networks

# Part IX

**Discrete** choice

### **Discrete choice**

### 15.1 Random utility

### 15.1.1 Continuous utility functions

With non-discrete choice a consumer chooses how much of product x to consume.

The utility function is of the form:

 $U_i(x_1, \dots, x_m; d)$ 

And the consumer chooses how much of  $x_i$  to consume to maximise this, subject to the budget constraint.

We include features relating to the individual, d.

### 15.1.2 Discrete choice

The utility customer i gets from product j is:

 $U_{ij} = f_i(p,d) + \epsilon_{ij}$ 

The customer chooses the product with the highest utility.

### 15.1.3 The outside option

We need to know the market share of the outside option. Do this theoretically. Eg number of customers in area, and 1 per day.

### 15.2 Linear random utility functions

### 15.2.1 Price preferences

We start with a simple model, where the customer has price preference.

 $U_{ij} = -\beta_i p_{ij} + \epsilon_{ij}$ 

#### 15.2.2 Product characteristics

 $U_{ij} = \alpha_i x_j - \beta_i p_{ij} + \epsilon_{ij}$ 

### 15.2.3 Individual characteristics

 $U_{ij} = \alpha_i x_j - \beta_i p_{ij} + \theta_j d_i + \epsilon_{ij}$ 

### 15.2.4 The general form

We can convert this to the form:

 $U_{ij} = \Theta z_{ij} + \epsilon_{ij}$ 

# 15.3 Modelling homogeneous preferences with multinomial logit

15.3.1 Modelling homogeneous preferences with multinomial logit

### 15.3.2 Recap

Our model is:

 $U_{ij} = \Theta z_{ij} + \epsilon_{ij}$  $U_{ij} = \alpha_i x_j - \beta_i p_{ij} + \theta_j d_i + \epsilon_{ij}$ 

#### 15.3.3 Homogeneous model

We model all customers as having the same preferences.

 $U_{ij} = \alpha x_j - \beta p_{ij} + \theta_j d_i + \epsilon_{ij}$ 

### 15.3.4 The multinomial logit assumption

If errors are IID and extreme we get:

$$P_{ij} = \frac{e^{\Theta z_j}}{\sum_k e^{\Theta z_k}}$$

### 15.3.5 The outside option

A user has the option of not buying anything.

 $U_0 = 0$ 

This gives us the following shares:

$$P_{ij} = \frac{e^{\Theta z_j}}{e^0 + \sum_{k=1} e^{\Theta z_k}}$$
$$P_{ij} = \frac{e^{\Theta z_j}}{1 + \sum_{k=1} e^{\Theta z_k}}$$

### 15.3.6 Own-price elasticity of demand

$$P_{ij} = \frac{e^{\Theta z_j}}{1 + \sum_{k=1}} e^{\Theta z_k}$$

$$P_{ij} = \frac{e^{\alpha x_j - \beta p_j + \theta_j d_i}}{1 + \sum_{k=1} e^{\alpha x_k - \beta p_k + \theta_k d_i}}$$

$$\frac{\delta P_{ij}}{\delta p_j} \frac{p_j}{P_{ij}} = -\beta p_j (1 - P_{ij})$$

$$\frac{\delta P_{ij}}{\delta p_k} \frac{p_j}{P_{ij}} = \beta p_k P_{ij}$$

This means that the lower the price, the lower the own price elasticity of demand.

This means that mark ups are higher for cheaper goods, which doesn't always match reality.

This can be adjusted by changing the form. For example we could use  $\ln p$  or  $p^2.$ 

However, we are still getting the shape by assumption.

### 15.3.7 Cross-price elasticity of demand

### 15.3.8 Getting aggregate market shares

$$s_j = \frac{1}{n} \sum_i P_{ij}$$

# 15.4 Modelling homogeneous preferences with nested logit

### 15.4.1 Nested logit

With IID all goods are equal substitutes. If prices rise customers will switch to others in proportion to market size.

In practice if the price of a cheap car rises, there will be more substitution to other cheap cars than expensive cars.

Nested logit can address this.

### 15.4.2 Estimating the nested logit model

As with the multinomial logit model, we have prices, market shares and product characteristics.

#### 15.4.3 Own-price elasticity of demand

15.4.4 Cross-price elasticty of demand

### 15.5 Modelling heterogeneous preferences with individual characteristics

#### 15.5.1 Recap

In the multinomial logit model we had:

 $P_{ij} = \frac{e^{\Theta z_j}}{1 + \sum_{k=1} e^{\Theta z_k}}$ 

### 15.5.2 Adding customer characteristics

If  $z_i$  just includes product characteristics then we have homogeneous preferences.

We can include customer level data in  $z_j$ , for example individual income, location, age etc.

### 15.5.3 Estimation

As before, we want product characteristics and prices.

However rather than market share we instead use customer level information.

# 15.6 Modelling heterogeneous preferences with mixed logit

# 15.6.1 Modelling heterogeneous preferences with mixed logit

Using demographic data

# Discrete choice estimation using aggregate market data

### 16.1 Discrete choice estimation

- 16.1.1 Discrete choice estimation with aggregate data
- 16.1.2 Discrete choice estimation with individual data

### 16.1.3 Omitted variable bias

If the producer sees customer characteristics we do not, then there will be a bias in our estimate.

Producers will set prices correlated with those characteristics.

#### 16.1.4 The problem with alternative price data

We need alternative price data for this multinomial choice model.

These are not observed, and so we need to estimate them.

One option is to use list prices, or average prices, for all prices.

However, differences from this are likely correlated with individual characteristics, giving us bias.

### 16.1.5 Dummy variables for products

By adding a dummy for each product we control for unobserved variables. This adds a parameter for each product, which can increase the variance of the estimates. Adding the dummies affects the other estimators. We can fix this using minimum distance estimator. The error no longer includes unobserved characteristics.

Dummies for groups. These create fewer new parameters.

### 16.1.6 Using instrumental variables

- 16.2 Other
- 16.2.1 BLP demand curve estimation (Berry, Levinsohn, Pakes)

# Discrete choice estimation using customer level data

17.1 Discrete choice estimation with individual data