Complex analysis

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Part I

Complex analysis

Chapter 1

Complex calculus

1.0.1 Complex-valued functions

1.0.2 Defining complex valued functions

We can consider complex valued functions as a type of vector fields.

1.0.3 Line integral of the complex plane

$\int_C f(r)ds = \lim_{\Delta srightarrow0} \sum_{i=0}^n f(r(t_i))\Delta s_i$

$\int_C f(r)ds = \lim_{\Delta srightarrow0} \sum_{i=0}^n f(r(t_i))\frac{\delta r(t_i)}{\delta t}\delta r_i$

$\int_C f(z)dz = \int_a^b f(r(t_i))\frac{\delta r(t_i)}{\delta t}\delta r_i$

1.0.4 Complex continuous functions

1.0.5 Open regions

1.0.6 Analytic continuation

1.0.7 Analytic functions

1.0.8 Circle of convergence

1.0.9 Complex differentiation

1.0.10 Wirtinger derivatives

Previously we had partial differentiation on the real line. We could use the partial differention operator

We want to find a similar operator for the complex plane.

1.0.11 Line integral of the complex plane

$\int_C f(r)ds = \lim_{\Delta s rightarrow 0} \sum_{i=0}^{n} f(r(t_i))\Delta s_i$

$\int_C f(r)ds = \lim_{\Delta s rightarrow 0} \sum_{i=0}^{n} f(r(t_i))\frac{\delta r(t_i)}{\delta t}\delta r_i$

$\int_C f(z)dz = \int_a^b f(r(t_i))\frac{\delta r(t_i)}{\delta t}\delta r_i$

1.0.12 Complex integration

1.0.13 Complex smooth functions

If a function is complex differentiable, it is smooth.

1.0.14 All differentiable complex functions are smooth

1.0.15 All smooth complex functions are analytic

1.0.16 Singularities

1.0.17 Contour integration

1.0.18 Line integral

1.0.19 Cauchy's integral theorem

1.0.20 Cauchy's integral formula

1.0.21 Cauchy-Riemann equations

Consider complex number z=x+iy

A function on this gives:

$f(z) = u + iv$

Take the total differential of :

$df/dz = \frac{\delta f}{\delta z} + \frac{\delta f}{\delta x}\frac{dx}{dz} + \frac{\delta f}{\delta y}\frac{dy}{dz}$

We know that:

- $\frac{dx}{dz} = 1$
- $\frac{dy}{dz} = -i$

We can see from this that

- $\frac{du}{dx} = \frac{dv}{dy}$

- $\frac{du}{dy} = -\frac{dv}{dx}$

These are the Cauchy-Riemann equations

Chapter 2

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Quadratic maps, the logistic map, the Mandlebrot set, the Julia set and the Newton fractal

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