Quantum mechanics and quantum computing

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December 15, 2025

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Preface

This is a live document, and is full of gaps, mistakes, typos etc.

Part I

Quantum mechanics

Chapter 1

Quantum mechanics

1.1 Pure quantum states

1.1.1 Discrete states as vectors

1.1.2 Observables as linear operators

1.1.3 Orthonormal basis

1.1.4 Constructing a Hermitian matrix for an observable

1.1.5 Spin of a single particle

1.2 Mixed quantum states

1.2.1 Mixed quantum states

1.2.2 Probability amplitudes

1.2.3 Probability

1.3 State evolution

1.3.1 Indexing states to time

We have state defined at each time t.

$\Psi(t)$.

1.3.2 Wave functions

We have state $\Psi(t)$.

$\psi(x,t) = \langle x|\Psi(t)\rangle$

This is the wave function.

1.3.3 Schrodinger

Discrete time

With discrete time we can use a canonical operator for moving between discrete states in single jumps.

With discrete time there must a countable number of states.

We can index time to the integers.

At time 0 we have v

At time 1 we have Ψv

At time 2 we have $\Psi\Psi v$

We can write this as $\Psi(t_1, t_0) = \Psi^{t_1 - t_0}$

1.3.4 Representation theory for the time group

Time is a linear operator

Instead, we describe the time operator as a Lie group, using Lie algebra.

$\Psi(t_b - t_a) = e^{(t_b - t_a)X}$

States are vectors

We can remove a degree of freedom by using norm of 1 for vectors

For each dynamic system we define a set of possible states.

We can describe a state $v \in V$.

Finite state spaces

We can describe a system like heads or tails.

Infinite state spaces

This can describe continous position, or an angle.

1.3.5 Indexing time to the real numbers

Sloan's theorem

1.3.6 Continous time with Lie algebra

We use $X = iH$, what are the implications of this compared to other choices?

Lie algebras with $n \times x$

This loops back? multiple dimensions, infinite, so maybe not?

With continuous time we do not have a single operator to describe movements. There is always one smaller.

With continous time there must be either a single state, or an uncountably infinite number of states.

$U = M_n^n$

$U = (I + \frac{1}{n}G_n)^n$

$U = \lim_{n\to\infty}(I + \frac{1}{n}G)^n$

Now:

$UU^* = I$

$(I + \frac{1}{n}G)(I + \frac{1}{n}G)^* = I$

$(I + \frac{1}{n}G)(I + \frac{1}{n}G^*) = I$

$G = -G^*$

$G = iH$

$iH = -(iH)^*$

$H = H^*$

H is Hermitian

$U = \lim_{n\to\infty}(I + \frac{1}{n}iH)^n$

This isn't quite right, need defined for different time jumps.

1.3.7 Unitary time

Why? What's the interpretation here? Is this an assumption, or just a modelling choice?

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)X^*}e^{(t_b-t_a)X}$

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)(X^*+X)}$

$X = iH$

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)(-iH+iH)} = I$

$\Psi(t_b - t_a) = e^{(t_b - t_a)iH}$

1.3.8 The time-depedendent general Schrödinger equation

$v(t_b) = e^{(t_b - t_a)X} v(t_a)$

$v(t + \delta) = e^{\delta X} v(t)$

$v(t + \delta) = (I + \delta X) v(t)$

$$\frac{v(t + \delta) - v(t)}{\delta} = Xv(t)$$

$$\frac{\delta v(t)}{\delta t} = Xv(t)$$

$$\frac{\delta v(t)}{\delta t} = iHv(t)$$

1.3.9 The energy operator and the time-indepedendent general Schrödinger equation

$$E = ih\frac{\delta}{\delta t}$$

$Ev(t) = Hv(t)$

1.4 Infinite dimensional quantum states

1.4.1 Position

1.4.2 Velocity

1.4.3 Momentum

1.4.4 Moving to 3 dimensions

1.4.5 The action integral

1.4.6 Renormalisation

1.5 Quantum entanglement

1.6 Other

1.6.1 The Hamiltonian of quantum mechanics

1.6.2 Plank's constant

We can add Plank's constant, due to the arbitrary scaling of time.

1.6.3 Phase shift

1.6.4 Density matrix

1.6.5 Born rule

1.6.6 Spin-statistics theorem

1.6.7 Heisenberg's uncertainty principle

Result of spin-statistics theorem?

1.6.8 The Dirac equation

1.6.9 The quantum harmonic oscillator

Chapter 2

The Heisenberg picture of quantum mechanics

2.1 Introduction

Part II

Relativistic quantum mechanics

Chapter 3

Relativistic quantum mechanics

3.1 Introduction

3.1.1 Introduction

It typically deals with wave equations, like the Dirac equation for fermions and the Klein-Gordon equation for bosons, that are consistent with both quantum mechanics and special relativity. These equations account for phenomena such as particles having spin and the existence of antiparticles.

Part III

Quantum Field Theory (QFT)

Chapter 4

Quantum Field Theory (QFT)

4.1 Introduction

Part IV

Other

Chapter 5

The hydrogen atom

5.1 Introduction

5.1.1 Atomic states

transition matrix can be v complex

5.1.2 Factored states

+ can store rules for simply

finite atomic: permutation matrix inf atomic: how to represent?

basis state dynamic to comp sci??

factored: how to model atomic as factored?

Part V

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Chapter 6

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6.1 Introduction

6.1.1 Introduction

Gleason's theorem can be used to derive born rule?

can we Model discrete physics as moving between eigenstates with unitary operators?

quantum mechanics now describes state as v, and then also Psi as state, and then defines Psi.v seems inconsistent?

physics big page: explicitly say qed, electroweak, quantum chromodynamics

quantum mechanics before qed non relativistic particles have wave like properties deriving born rule from many worlds qed virtual particles particles are excitation of underlying field renormalisation charge symmetry parity symmetry time symmetry chirality and helicity muon: electron but higher mass tau: higher mass again

neutrios: neutino (electron) muon neutrino (muon) tau neutrino (tau) why these number of electric particles? Unclear why only 3? Unclear why only -1,0,1. (reasons exist but complex?) parity symmetry exists energy density of empty space? fermion: half integer spin lepton: fermion without color

electroweak spontaneous symmetry breaking superconductivity? Related to breaking U(1), not sure if part of electroweak, or QED, or requires atoms therefore QCD? no parity symmetry. Gives way to define handedness of universe? charge symmetry? cp symmetry cp violations happens for some electroweak phenomena? cpt symmetry adds new gauge bosons. Qed just has photons $W\hat{+}$, $W\hat{-}$, $Z\hat{0}$ gauge bosons have mass, unlike photon mediate weak force higgs boson and higgs field doesn't predict why specific masses we see of particles though strong no cp violations? Unclear why, note there are for electroweak page on reconsiling quantum with general vacuum catastrophe singularities hawking radiation

Quantum systems Set of possible values Use these as eigenvalues and construct hermitian matrix State is vector. Vector being equal to eigen vector means at eigen state. Orthogonal states because only 1 at a time Do that on classical discrete before moving to quantum

quantum spin statistics theorem bose-einstein statistics (integer spin) fermi-dirac statistics (half integer spin)

concept of spin comes from quantum electromagnetism. spinning large object generates field in direction. so does electron

Quantum mechanics If discrete states sum of states, if continuous integral and coefficient is wave function

nuclear physics + nuclear binding energy + radioactive decay + exponential decay + half life + fusion + fission + fissile isotope. most common is 235U + enrichment. making ore into 235U from mix of that and 238U + critial mass decreses with more purity + control rods: absorb material + prompt critical neutrons vs delayed critical neutrons + effective neutron multiplication factor

number of pure states is the dimension of state vector. if there are 2 states, represent as 2 vector with 0 and 11.

for pauli matrices. start with one, then construct others such that prob = 0.5. so 2 state system results in 3 vector measurements. 3 dimensions

de broglie wavelength

link between hermit matrix and eigen vec val real?

6.1.2 quantum computing (h3 for computers)

on quantum computing: quantum fourier transform. quantum equivalent of discrete fourier transform. exponentially faster than discrete. quantum error correction in quantum computing. Quantum: Grover algorithm

quantum computers + shor's algorithm + postBQP (BQP with post selection) + bounded-error polynomial time (BQP) (1/3 cut off) + Quantum Merlin Arthur (QMA) verify on quantum computer in poly time

quantum computing + turing machines can do all things quantum can do + but possibly different complexity classes + qubits

page on quantum computing? + classical memory: series of bits + quantum memory: superposition of classical bits + what do we do with these states? in classical apply logical gates. now, apply quantum logic gates + measuring state: in classical, simple, deterministic. in quantum, not. want high probability of correct answer + types of quantum gates: in classical, can create any using and, not, etc. what is equivalent for quantum? + measurement happens at end. otherwise interupts