Quantum mechanics and quantum computing

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Quantum mechanics

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1.2.1 Mixed quantum states

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1.2.3 Probability

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1.3.1 Indexing states to time

We have state defined at each time t.

$\Psi(t)$.

1.3.2 Wave functions

We have state $\Psi(t)$.

$\psi(x,t) = \langle x|\Psi(t)\rangle$

This is the wave function.

1.3.3 Schrodinger

Discrete time

With discrete time we can use a canonical operator for moving between discrete states in single jumps.

With discrete time there must a countable number of states.

We can index time to the integers.

At time 0 we have v

At time 1 we have Ψv

At time 2 we have $\Psi\Psi v$

We can write this as $\Psi(t_1, t_0) = \Psi^{t_1 - t_0}$

1.3.4 Representation theory for the time group

Time is a linear operator

Instead, we describe the time operator as a Lie group, using Lie algebra.

$\Psi(t_b - t_a) = e^{(t_b - t_a)X}$

States are vectors

We can remove a degree of freedom by using norm of 1 for vectors

For each dynamic system we define a set of possible states.

We can describe a state $v \in V$.

Finite state spaces

We can describe a system like heads or tails.

Infinite state spaces

This can describe continous position, or an angle.

1.3.5 Indexing time to the real numbers

Sloan's theorem

1.3.6 Continous time with Lie algebra

We use $X = iH$, what are the implications of this compared to other choices?

Lie algebras with $n \times x$

This loops back? multiple dimensions, infinite, so maybe not?

With continuous time we do not have a single operator to describe movements. There is always one smaller.

With continous time there must be either a single state, or an uncountably infinite number of states.

$U = M_n^n$

$U = (I + \frac{1}{n}G_n)^n$

$U = \lim_{n\to\infty}(I + \frac{1}{n}G)^n$

Now:

$UU^* = I$

$(I + \frac{1}{n}G)(I + \frac{1}{n}G)^* = I$

$(I + \frac{1}{n}G)(I + \frac{1}{n}G^*) = I$

$G = -G^*$

$G = iH$

$iH = -(iH)^*$

$H = H^*$

H is Hermitian

$U = \lim_{n\to\infty}(I + \frac{1}{n}iH)^n$

This isn't quite right, need defined for different time jumps.

1.3.7 Unitary time

Why? What's the interpretation here? Is this an assumption, or just a modelling choice?

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)X^*}e^{(t_b-t_a)X}$

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)(X^*+X)}$

$X = iH$

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)(-iH+iH)} = I$

$\Psi(t_b - t_a) = e^{(t_b - t_a)iH}$

1.3.8 The time-depedendent general Schrödinger equation

$v(t_b) = e^{(t_b - t_a)X} v(t_a)$

$v(t + \delta) = e^{\delta X} v(t)$

$v(t + \delta) = (I + \delta X)v(t)$

$\frac{v(t+\delta) - v(t)}{\delta} = Xv(t)$

$\frac{\delta v(t)}{\delta t} = Xv(t)$

$\frac{\delta v(t)}{\delta t} = iHv(t)$

1.3.9 The energy operator and the time-indepedendent general Schrödinger equation

$E = ih\frac{\delta}{\delta t}$

$Ev(t) = Hv(t)$

1.4 Infinite dimensional quantum states

1.4.1 Position

1.4.2 Velocity

1.4.3 Momentum

1.4.4 Moving to 3 dimensions

1.4.5 The action integral

1.4.6 Renormalisation

1.5 Quantum entanglement

1.6 Other

1.6.1 The Hamiltonian of quantum mechanics

1.6.2 Plank's constant

We can add Plank's constant, due to the arbitrary scaling of time.

1.6.3 Phase shift

1.6.4 Density matrix

1.6.5 Born rule

1.6.6 Spin-statistics theorem

1.6.7 Heisenberg's uncertainty principle

Result of spin-statistics theorem?

1.6.8 The Dirac equation

1.6.9 The quantum harmonic oscillator

Chapter 2

Quantum Field Theory (QFT)

2.1

Chapter 3

The hydrogen atom

3.1 Introduction

3.1.1 Atomic states

transition matrix can be v complex

3.1.2 Factored states

+ can store rules for simply

finite atomic: permutation matrix inf atomic: how to represent?

basis state dynamic to comp sci??

factored: how to model atomic as factored?