Physics

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Preface

This is a live document, and is full of gaps, mistakes, typos etc.

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Chapter 1

Newtonian mechanics

1.1 Introduction

1.1.1 SUVAT

Introduction

For a constant acceleration environment we want to find equations to link:

- Initial speed: v_{t_0}
- End speed: v_{t_1}
- Time: $t_1 - t_0$
- Acceleration: a
- Displacement $s_{t_1} - s_{t_0}$

The SUVAT equations

These are the following, and are derived below.

- $v_{t_1} = a(t_1 - t_0) + v_{t_0}$
- $(s_{t_1} - s_{t_0}) = v_{t_0}(t_1 - t_0) + \frac{1}{2}a(t_1 - t_0)^2$
- $(s_{t_1} - s_{t_0}) = v_{t_1}(t_1 - t_0) - \frac{1}{2}a(t_1 - t_0)^2$
- $v_{t_1}^2 = v_{t_0}^2 + 2a(s_{t_1} - s_{t_0})$

- $(s_{t_1} - s_{t_0}) = (t_1 - t_0)\frac{v_{t_1} + v_{t_1}}{2}$

Equation 1: No displacement

This equation is:

$v_{t_1} = a(t_1 - t_0) + v_{t_0}$

To derive this start with:

$v_t := \frac{\delta s_t}{\delta t}$

$a := \frac{\delta v_t}{\delta t}$

If acceleration is constant, then

$\frac{\delta v_t}{\delta t} = a$

$v_t = \int a dt + v_0$

$v_t = at + v_0$

Equation 2: No end velocity

This equation is:

$(s_{t_1} - s_{t_0}) = v_{t_0}(t_1 - t_0) + \frac{1}{2}a(t_1 - t_0)^2$

To derive this start with:

$v := \frac{\delta s_t}{\delta t}$

Then:

$\frac{\delta s_t}{\delta t} = at + v_0$

$s_t = \frac{1}{2}at^2 + v_0 t + s_0$

$(s_t - s_0) = v_0 t + \frac{1}{2}at^2$

Equation 3: No start velocity

This equation is:

$(s_{t_1} - s_{t_0}) = v_{t_1}(t_1 - t_0) - \frac{1}{2}a(t_1 - t_0)^2$

To derive this start with:

$v_t = at + v_0$

$(s_t - s_0) = t\frac{v_t + v_0}{2}$

So:

$v_0 = v_t - at$

$v_0 = \frac{2}{t}(s_t - s_0) - v_t$

$v_t - at = \frac{2}{t}(s_t - s_0) - v_t$

$(s_t - s_0) = v_t t - \frac{1}{2}at^2$

Equation 4: No time

This equation is:

$v_{t_1}^2 = v_{t_0}^2 + 2a(s_{t_1} - s_{t_0})$

To derive this start with:

$v_t = at + v_0$

$(s_t - s_0) = t\frac{v_t + v_0}{2}$

So:

$t = \frac{v_t - v_0}{a}$

$t = 2\frac{s_t - s_0}{v_t + v_0}$

$\frac{v_t - v_0}{a} = 2\frac{s_t - s_0}{v_t + v_0}$

$(v_t - v_0)(v_t + v_0) = 2a(s_t - s_0)$

$v_t^2 = v_0^2 + 2a(s_t - s_0)$

Equation 5: No acceleration

This equation is:

$(s_{t_1} - s_{t_0}) = (t_1 - t_0)\frac{v_{t_1} + v_{t_1}}{2}$

To derive this start with:

$v_t = at + v_0$

$$s_t - s_0 = \frac{1}{2}at^2 + v_0 t$$

So:

$$a = \frac{v_t - v_0}{t}$$

$$a = \frac{2[(s_t - s_0) - v_0 t]}{t^2}$$

$$\frac{v_t - v_0}{t} = \frac{2[(s_t - s_0) - v_0 t]}{t^2}$$

$$t(v_t - v_0) = 2[(s_t - s_0) - v_0 t]$$

$$t(v_t + v_0) = 2(s_t - s_0)$$

$$(s_t - s_0) = t\frac{v_t + v_0}{2}$$

Chapter 2

Paths

2.1 Describing paths

2.1.1 Describing events

In vector space $\mathbb{R}^n$.

$\mathbf{q} \in \mathbb{R}^n$

2.1.2 Describing the path of a particle

Also known as a worldline.

Index to t

$\mathbf{q}(t)$

2.1.3 Describing the velocity of a particle

$v = \frac{\delta \mathbf{q}}{\delta t}$

2.1.4 Describing the acceleration of a particle

$a = \frac{\delta v}{\delta t}$

$a = \frac{\delta^2 \mathbf{q}}{\delta t^2}$

2.2 Action

2.2.1 Action

We observe a particle moving in a path. We want to model the path that the particle takes.

The path is in a vector space, with coordinates $\mathbf{q}$. These coordinates could refer to the x, y, z and t coordinates we are are familar with.

For the path we have a start point a and end point b. We can define the length of the path as:

$S = \int_a^b d\tau$

We call S the action.

2.2.2 Linear metrics

We use a linear metric.

$\tau^2 = \mathbf{q}^T \mathbf{M} \mathbf{q}$

So:

$d\tau^2 = (d\mathbf{q})^T \mathbf{M} d\mathbf{q}$

$S = \int_a^b \sqrt{(d\mathbf{q})^T \mathbf{M} d\mathbf{q}}$

2.2.3 Time and velocity

$S = \int_a^b \sqrt{\frac{1}{dt^2}(d\mathbf{q})^T \mathbf{M} d\mathbf{q}} dt$

$S = \int_a^b \sqrt{(\frac{d\mathbf{q}}{dt})^T \mathbf{M} \frac{d\mathbf{q}}{dt}} dt$

$S = \int_a^b \sqrt{(\dot{\mathbf{q}})^T \mathbf{M} \dot{\mathbf{q}}} dt$

2.3 The Lagrangian

2.3.1 Lagrangians

We have:

$S = \int_a^b \sqrt{(\dot{\mathbf{q}})^T \mathbf{M} \dot{\mathbf{q}}} dt$

We can define:

$L = \sqrt{(\dot{\mathbf{q}})^T \mathbf{M}\dot{\mathbf{q}}}$

So we have:

$S = \int_a^b L dt$

2.3.2 Principle of stationary action

$\delta A = 0$

That is, the coordinates and their velocities are such that action is stationary.

2.3.3 Euler-Lagrange

We have $q(t)$ which makes the action stationary. Consider adding proportion ϵ of another function $f(t)$ to $q(t)$.

$A = \int_{t_0}^{t_1} L[q(t) + \epsilon f(t), q\dot{(t)} + \epsilon f(t)]dt$

$\frac{A - A}{\epsilon} = \frac{1}{\epsilon} \int_{t_0}^{t_1} L[q(t) + \epsilon f(t), q\dot{(t)} + \epsilon f(t)] - L[q, \dot{q}]dt$

We can do a Taylor expansion of A.

$A = \int_{t_0}^{t_1} L[q(t) + \epsilon f(t), q\dot{(t)} + \epsilon f(t)]dt$

$A = \int_{t_0}^{t_1} L[q(t), q\dot{(t)}] + \epsilon[f \frac{\delta L}{\delta q} + f\cdot \frac{\delta L}{\delta \dot{q}}] + \epsilon^2[...]dt$

So:

$\frac{A - A}{\epsilon} = \frac{1}{\epsilon} \int_{t_0}^{t_1} L[q(t), q\dot{(t)}] + \epsilon[f \frac{\delta L}{\delta q} + f\cdot \frac{\delta L}{\delta q\cdot}] + \epsilon^2[...] - L[q, \dot{q}]dt$

$\frac{A - A}{\epsilon} = \int_{t_0}^{t_1} [f \frac{\delta L}{\delta q} + f\cdot \frac{\delta L}{\delta \dot{q}}] + \epsilon[...]dt$

We can now make the left side 0, by using the definition of stationary action.

$\lim_{\epsilon \to 0} \frac{A - A}{\epsilon} = \int_{t_0}^{t_1} [f \frac{\delta L}{\delta q} + f\cdot \frac{\delta L}{\delta \dot{q}}]dt$

$\int_{t_0}^{t_1} [f \frac{\delta L}{\delta q} + f\cdot \frac{\delta L}{\delta \dot{q}}]dt = 0$

$\int_{t_0}^{t_1} [f \frac{\delta L}{\delta q}]dt + \int_{t_0}^{t_1} [f\cdot \frac{\delta L}{\delta \dot{q}}]dt = 0$

Note that

$\int_{t_0}^{t_1} [f\cdot \frac{\delta L}{\delta \dot{q}}]dt = [f \frac{\delta L}{\delta \dot{q}}]_{t_0}^{t_1} - \int_{t_0}^{t_1} f \frac{d}{dt} \frac{\delta L}{\delta \dot{q}} dt$

We assume that $f(t_0) = f(t_1) = 0$ and so:

$\int_{t_0}^{t_1} [f \cdot \frac{\delta L}{\delta \dot{q}}] dt = - \int_{t_0}^{t_1} f \frac{d}{dt} \frac{\delta L}{\delta \dot{q}} dt$

Plugging this back in we get:

$\int_{t_0}^{t_1} [f \frac{\delta L}{\delta q}] - f \frac{d}{dt} \frac{\delta L}{\delta \dot{q}} dt] = 0$

$\int_{t_0}^{t_1} f [\frac{\delta L}{\delta q}] - \frac{d}{dt} \frac{\delta L}{\delta \dot{q}} dt] = 0$

Since this applies to all possible functions we get:

$\frac{\delta L}{\delta q} = \frac{d}{dt} \frac{\delta L}{\delta \dot{q}}$

2.3.4 Definition: Momentum

$p = \frac{\delta L}{\delta \dot{q}}$

2.3.5 EL v2

$L = \sqrt{(\dot{\mathbf{q}})^T \mathbf{M} \dot{\mathbf{q}}}$

$\frac{\delta L}{\delta q} - \frac{d}{dt} (\frac{\delta L}{\delta \dot{q}}) = 0$

We have

$S = \int_a^b L(q(t), \dot{q}(t)) dt$

$\delta S = \delta \int_a^b L(q(t), \dot{q}(t)) dt$

$J = \int_a^b L(t, q(t), \dot{q}(t)) dt$

$J = \sum_{k=0}^{n-1}$

2.3.6 EL v3

$A = \sum L(x(t), \dot{x}(t)) \delta t$

$A = \sum L(\frac{x(t) + x(t-1)}{2}, \frac{x(t) - x(t-1)}{\delta t}) \delta t$

$\frac{\delta}{\delta x(t)} A = \sum \frac{\delta}{\delta x(t)} L(\frac{x(t) + x(t-1)}{2}, \frac{x(t) - x(t-1)}{\delta t}) \delta t$

$\frac{\delta}{\delta x(t)}A = \delta t[\frac{\delta}{\delta x(t)}L(\frac{x(t)+x(t-1)}{2}, \frac{x(t)-x(t-1)}{\delta t})+\frac{\delta}{\delta x(t)}L(\frac{x(t+1)+x(t)}{2}, \frac{x(t+1)-x(t)}{\delta t})]$

$\frac{\delta}{\delta x(t)}A = \delta t[\frac{1}{2}L_x + \frac{1}{\delta t}L_{\dot{x}} + \frac{1}{2}L_x - \frac{1}{\delta t}L_{\dot{x}}]$

$A = \int_a^b L(q(t), \dot{q}(t))dt$

$A = \sum L(q(t), \dot{q}(t))\delta t$

$A = \sum L(\frac{q(t)+q(t-1)}{2}, \frac{q(t)-q(t-1)}{\delta t})\delta t$

$\frac{\delta}{\delta q_i(t)}A = \sum \frac{\delta}{\delta q_i(t)}L(\frac{q(t)-q(t-1)}{2}, \frac{q(t)-q(t-1)}{\delta t})\delta t$

$\frac{\delta}{\delta q_i(t)}A = \delta t[\frac{\delta}{\delta q_i(t)}L(\frac{q(t)+q(t-1)}{2}, \frac{q(t)-q(t-1)}{\delta t})+\frac{\delta}{\delta q_i(t)}L(\frac{q(t+1)+q(t)}{2}, \frac{q(t+1)-q(t)}{\delta t})]$

$\frac{\delta}{\delta q_i(t)}A = \delta t[\frac{1}{2}L_{q_i}+\frac{1}{\delta t}L_{\dot{q}_i}+\frac{1}{2}L_{q_i}-\frac{1}{\delta t}L_{\dot{q}_i}+\frac{\delta}{\delta q_i(t)}L(\frac{q(t+1)+q(t)}{2}, \frac{q(t+1)-q(t)}{\delta t})]$

2.4 The Euclidian metric

2.4.1 The Euclidian metric

For the Euclidian metric:

$M = I$

$(dv)^T M dv = (dv)^T dv = dx^2 + dy^2 + dz^2$

$Action = \int \sqrt{dx^2 + dy^2 + dz^2}$

$Action = \int \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}dt$

$Action = \int v dt$

What are symmetries here? Gallilean group and?

2.4.2 Euclidian rotations

2.4.3 The Euclidan group

2.4.4 The Galilean group

2.5 Examples from Euler-Lagrange

2.5.1 Outcome

$\frac{\delta L}{\delta q} = \frac{d}{dt}\frac{\delta L}{\delta \dot{q}}$

$L = \sqrt{(\dot{\mathbf{q}})^T \mathbf{M}\dot{\mathbf{q}}}$

If $M = I$, then:

$L = \sqrt{(\dot{\mathbf{q}})^T \dot{\mathbf{q}}}$

In 1 dimension, Euclid:

$L = \sqrt{\dot{q}^2}$ $L = \dot{q}$

So:

$\frac{\delta L}{\delta q} = \frac{\delta}{\delta q}\dot{q}$ $\frac{\delta L}{\delta q} = 0$

$\frac{\delta L}{\delta \dot{q}} = \frac{\delta}{\delta \dot{q}}\dot{q}$ $\frac{\delta L}{\delta \dot{q}} = 1$

Into Euler-Lagrange:

$\frac{\delta L}{\delta q} = \frac{d}{dt}\frac{\delta L}{\delta \dot{q}}$ $0 = \frac{d}{dt}1$

In three dimensions:

$L = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$

So:

$\frac{\delta L}{\delta q} = \frac{\delta}{\delta q}\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ $\frac{\delta L}{\delta q} = 0$

$\frac{\delta L}{\delta \dot{q}} = \frac{\delta}{\delta \dot{q}}\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$

2.6 Other

2.6.1 Momentum

We define the momentum as:

$$p_j = \frac{\delta L}{\delta \dot{q}_j}$$

$$p_j = \frac{\delta}{\delta \dot{q}_j} \sqrt{(\dot{\mathbf{q}})^T \mathbf{M} \dot{\mathbf{q}}}$$

2.6.2 Force

Chapter 3

Symmetry

3.1 Introduction

3.1.1 Symmetry

We have a system of particles or something. we can do a measure on it.

We can do functions on the system.

What is preserved is an invariant measure

We observe event, worldline. other observers can:

3.1.2 Rotations

3.1.3 Translations

3.1.4 Boosts

Part here on adding velocities. show classical limit of normal adding them.

Chapter 4

Hamiltonian

4.1 Introduction

4.1.1 Legendre transformation

Chapter 5

The Lorentz metric

5.1 The Lorentz metric

5.1.1 The Lorentz metric

For lorentz:

$(\delta v)^T M \delta v = \delta t^2 - \delta x^2 - \delta y^2 - \delta z^2$

$Action = \int \sqrt{\delta t^2 - \delta x^2 - \delta y^2 - \delta z^2}$

$Action = \int \sqrt{1 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2} \delta t$

$Action = \int \sqrt{1 - v^2} \delta t$

5.1.2 The Lorentz metric with c

For lorentz with c

$(\delta v)^T M \delta v = \delta c^2 t^2 - \delta x^2 - \delta y^2 - \delta z^2$ p $Action = \int \sqrt{\delta c^2 t^2 - \delta x^2 - \delta y^2 - \delta z^2}$

$Action = \int \sqrt{1 - \frac{\dot{x}^2}{c^2} - \frac{\dot{y}^2}{c^2} - \frac{\dot{z}^2}{c^2}} c\delta t$

$Action = \int \sqrt{1 - \frac{v^2}{c^2}} c\delta t$

Because c is constant, we can simplify to:

$Action = \int \sqrt{1 - \frac{\dot{x}^2}{c^2} - \frac{\dot{y}^2}{c^2} - \frac{\dot{z}^2}{c^2}} \delta t$

$Action = \int \sqrt{1 - \frac{v^2}{c^2}} \delta t$

5.1.3 Lorentz rotations

5.1.4 Lorentz boosts

5.1.5 The Lorentz group

The Lorentz group consists of the Lorentz rotations and the Lorentz boosts.

5.1.6 The Poincar group

5.1.7 Group contraction from Lorentz to Euclid

5.1.8 Spacetime interval

5.1.9 Proper time

Chapter 6

Fields

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6.1.1 Fields

6.1.2 Action on a field

6.1.3 The Euler-Lagrange equations for fields

Part IV

Electromagnetism

Chapter 7

Electromagnetism

7.1 Introduction

7.1.1 Introduction

$F = qE + \nabla v M$

Part V

Statistical mechanics

Chapter 8

Statistical mechanics

8.1 Introduction

Part VI

Quantum mechanics

Chapter 9

Quantum mechanics

9.1 Pure quantum states

9.1.1 Discrete states as vectors

9.1.2 Observables as linear operators

9.1.3 Orthonormal basis

9.1.4 Constructing a Hermitian matrix for an observable

9.1.5 Spin of a single particle

9.2 Mixed quantum states

9.2.1 Mixed quantum states

9.2.2 Probability amplitudes

9.2.3 Probability

9.3 State evolution

9.3.1 Indexing states to time

We have state defined at each time t.

$\Psi(t)$.

9.3.2 Wave functions

We have state $\Psi(t)$.

$\psi(x,t) = \langle x|\Psi(t)\rangle$

This is the wave function.

9.3.3 Schrodinger

Discrete time

With discrete time we can use a canonical operator for moving between discrete states in single jumps.

With discrete time there must a countable number of states.

We can index time to the integers.

At time 0 we have v

At time 1 we have Ψv

At time 2 we have $\Psi\Psi v$

We can write this as $\Psi(t_1, t_0) = \Psi^{t_1-t_0}$

9.3.4 Representation theory for the time group

Time is a linear operator

Instead, we describe the time operator as a Lie group, using Lie algebra.

$\Psi(t_b - t_a) = e^{(t_b-t_a)X}$

States are vectors

We can remove a degree of freedom by using norm of 1 for vectors

For each dynamic system we define a set of possible states.

We can describe a state $v \in V$.

Finite state spaces

We can describe a system like heads or tails.

Infinite state spaces

This can describe continous position, or an angle.

9.3.5 Indexing time to the real numbers

Sloan's theorem

9.3.6 Continous time with Lie algebra

We use $X = iH$, what are the implications of this compared to other choices?

Lie algebras with $n \times x$

This loops back? multiple dimensions, infinite, so maybe not?

With continuous time we do not have a single operator to describe movements. There is always one smaller.

With continous time there must be either a single state, or an uncountably infinite number of states.

$U = M_n^n$

$U = (I + \frac{1}{n}G_n)^n$

$U = \lim_{n\to\infty}(I + \frac{1}{n}G)^n$

Now:

$UU^* = I$

$(I + \frac{1}{n}G)(I + \frac{1}{n}G)^* = I$

$(I + \frac{1}{n}G)(I + \frac{1}{n}G^*) = I$

$G = -G^*$

$G = iH$

$iH = -(iH)^*$

$H = H^*$

H is Hermitian

$U = \lim_{n\to\infty}(I + \frac{1}{n}iH)^n$

This isn't quite right, need defined for different time jumps.

9.3.7 Unitary time

Why? What's the interpretation here? Is this an assumption, or just a modelling choice?

$\Psi(t_b - t_a)^* \Psi(t_b - t_a) = e^{(t_b - t_a)X^*} e^{(t_b - t_a)X}$

$\Psi(t_b - t_a)^* \Psi(t_b - t_a) = e^{(t_b - t_a)(X^* + X)}$

$X = iH$

$\Psi(t_b - t_a)^* \Psi(t_b - t_a) = e^{(t_b - t_a)(-iH + iH)} = I$

$\Psi(t_b - t_a) = e^{(t_b - t_a)iH}$

9.3.8 The time-depedendent general Schrdinger equation

$v(t_b) = e^{(t_b - t_a)X} v(t_a)$

$v(t + \delta) = e^{\delta X} v(t)$

$v(t + \delta) = (I + \delta X)v(t)$

$\frac{v(t+\delta) - v(t)}{\delta} = Xv(t)$

$\frac{\delta v(t)}{\delta t} = Xv(t)$

$\frac{\delta v(t)}{\delta t} = iHv(t)$

9.3.9 The energy operator and the time-indepedendent general Schrdinger equation

$E = ih\frac{\delta}{\delta t}$

$Ev(t) = Hv(t)$

9.4 Infinite dimensional quantum states

9.4.1 Position

9.4.2 Velocity

9.4.3 Momentum

9.4.4 Moving to 3 dimensions

9.4.5 The action integral

9.4.6 Renormalisation

9.5 Quantum entanglement

9.6 Other

9.6.1 The Hamiltonian of quantum mechanics

9.6.2 Plank's constant

We can add Plank's constant, due to the arbitrary scaling of time.

9.6.3 Phase shift

9.6.4 Density matrix

9.6.5 Born rule

9.6.6 Spin-statistics theorem

9.6.7 Heisenberg's uncertainty principle

Result of spin-statistics theorem?

9.6.8 The Dirac equation

9.6.9 The quantum harmonic oscillator

Chapter 10

Quantum Field Theory (QFT)

10.1

Chapter 11

The hydrogen atom

11.1 Introduction

11.1.1 Atomic states

transition matrix can be v complex

11.1.2 Factored states

+ can store rules for simply

finite atomic: permutation matrix inf atomic: how to represent?

basis state dynamic to comp sci??

factored: how to model atomic as factored?

Part VII

Chemistry

Chapter 12

Atoms

Chapter 13

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Chapter 14

Acid

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Strong chemical bonds

15.1 Introduction

15.1.1 Covalent bonds

15.1.2 Ionic bonds

15.1.3 Metallic bonding

Electrons are shared between a large number of atoms

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Intermolecular bonds

Chapter 17

Isomers

Part VIII

Geology

Chapter 18

Geology

18.1 Introduction

18.1.1 Carbon cycle

Wood is renewable, but over what period? Replanting takes time.